Lecture 23

Introduction to Dynamic Programming

Source: Introduction to Algorithms, CLRS

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Idea: Instead of solving a subproblem **repeatedly**, solve it **once** and **store** the result in an array.

Let's learn DP through an example!

Rod Cutting

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Profit earned:	9	9

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Possible cuttings for a length 4 rod:	(4)	(1,3)	(2,2)
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Maximum profit

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Time Complexity: You can either cut or not cut at every ith inch. Generating all cuttings this way can lead to $O(2^n)$ time.

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For the remaining $(n - i)$ length rod,
we cannot get more than $profit_{n-i}$.

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return $profit$

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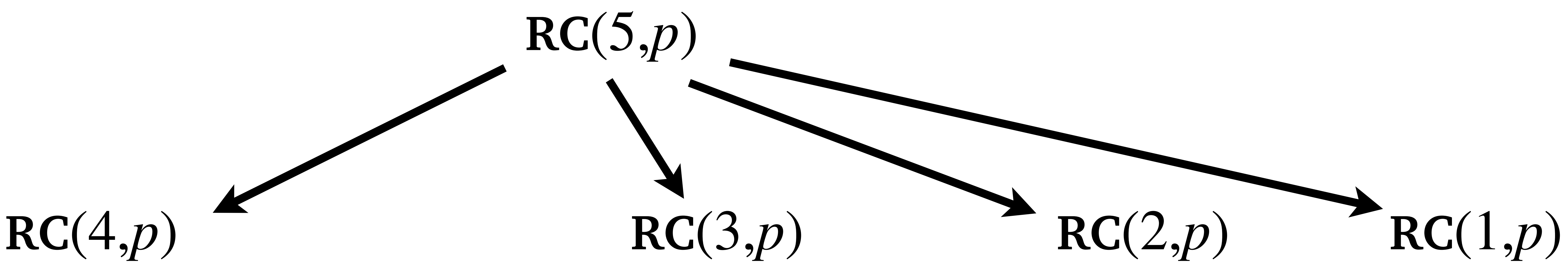


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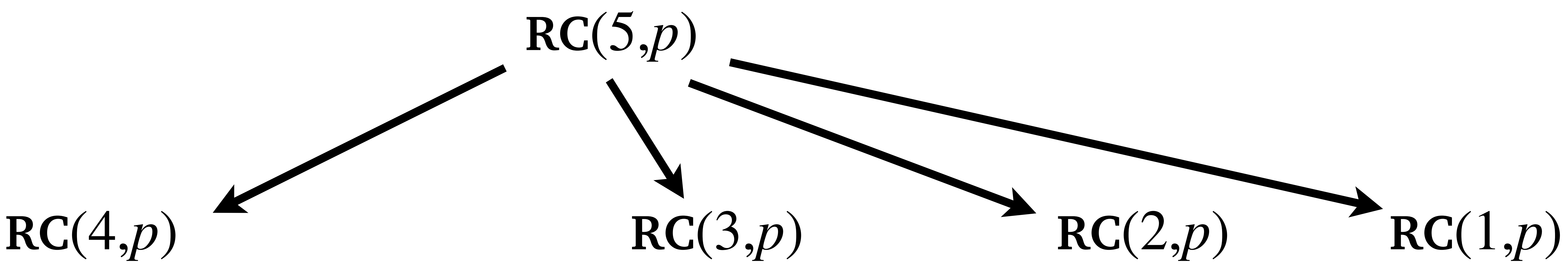

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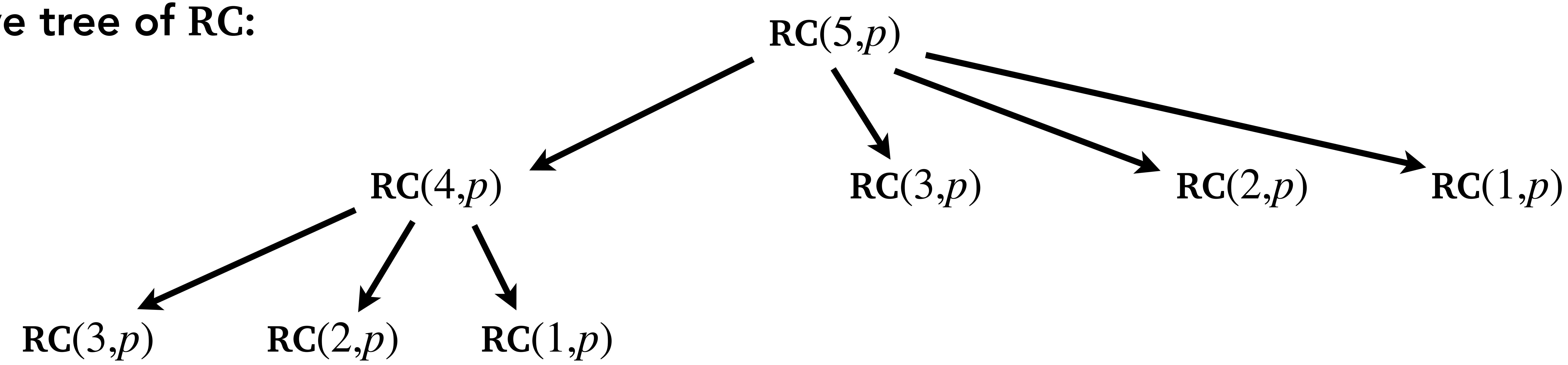

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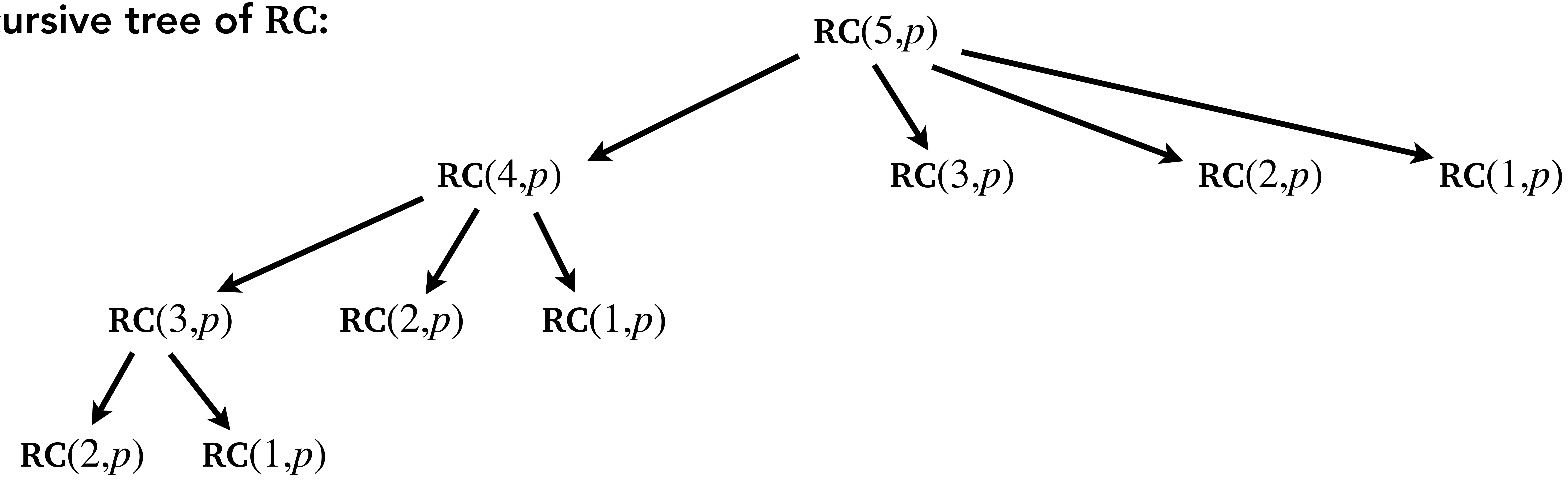

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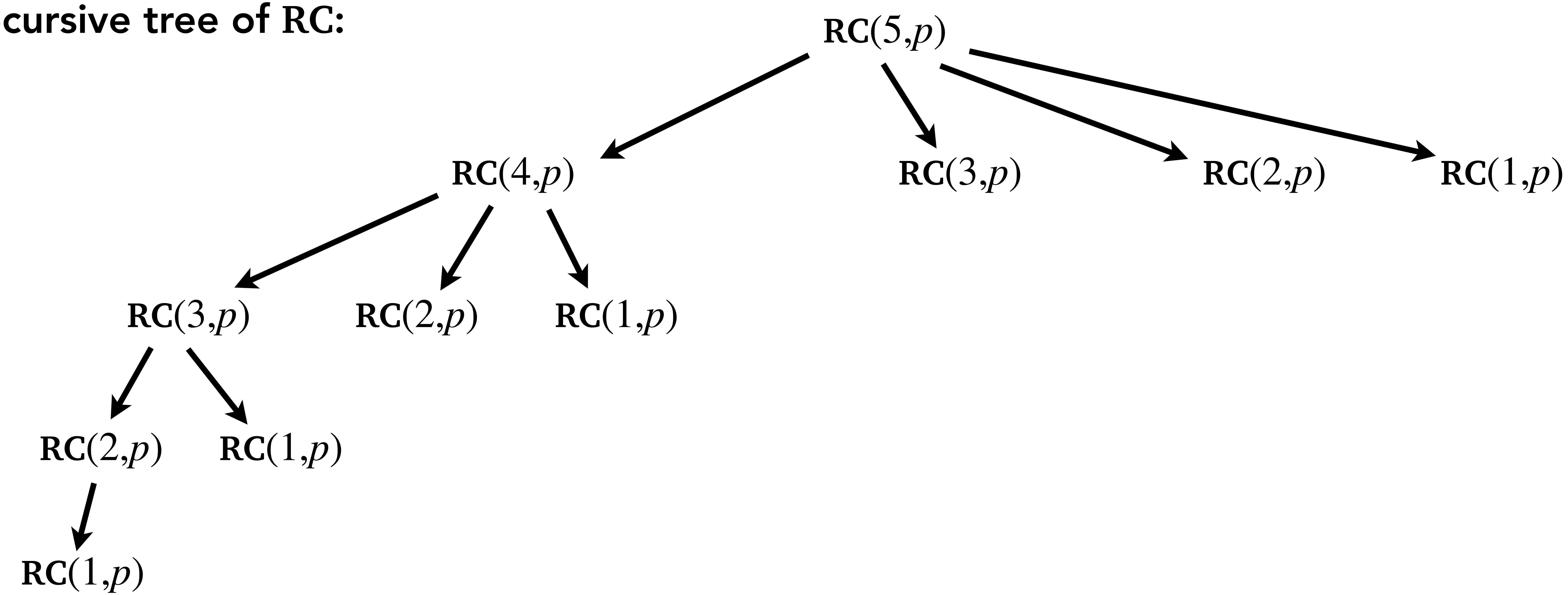

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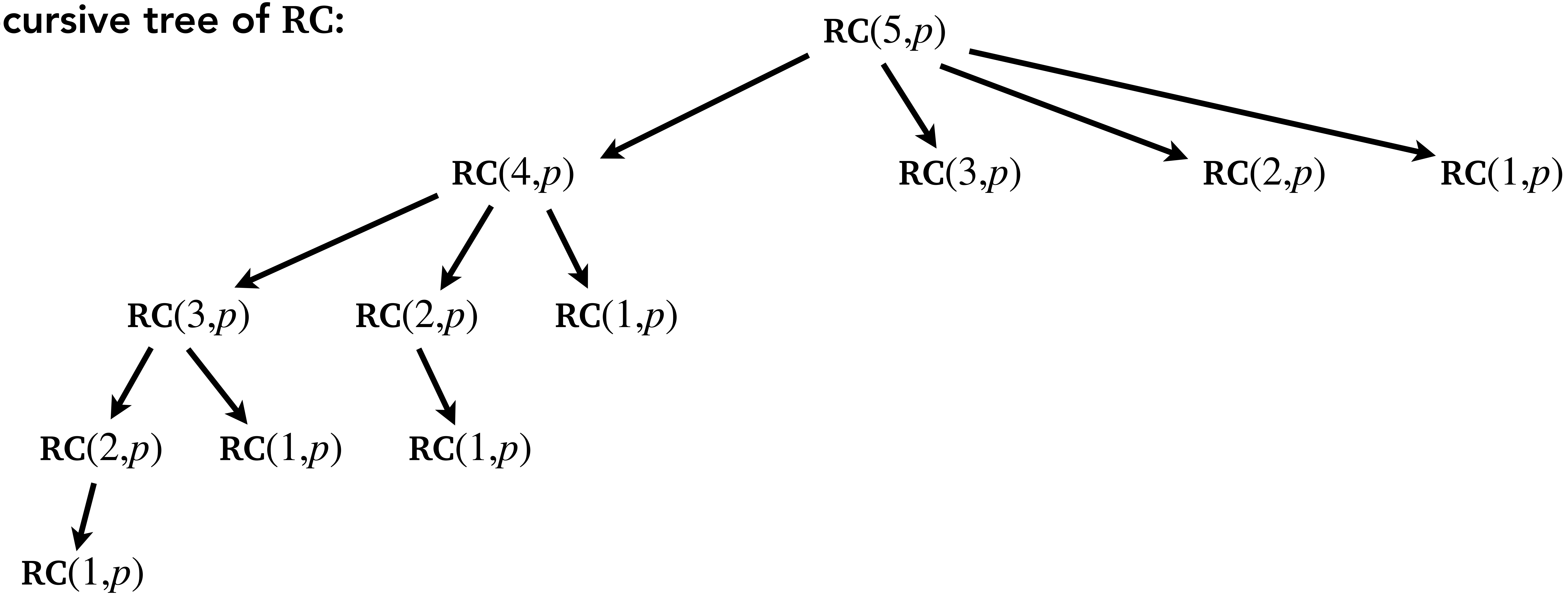

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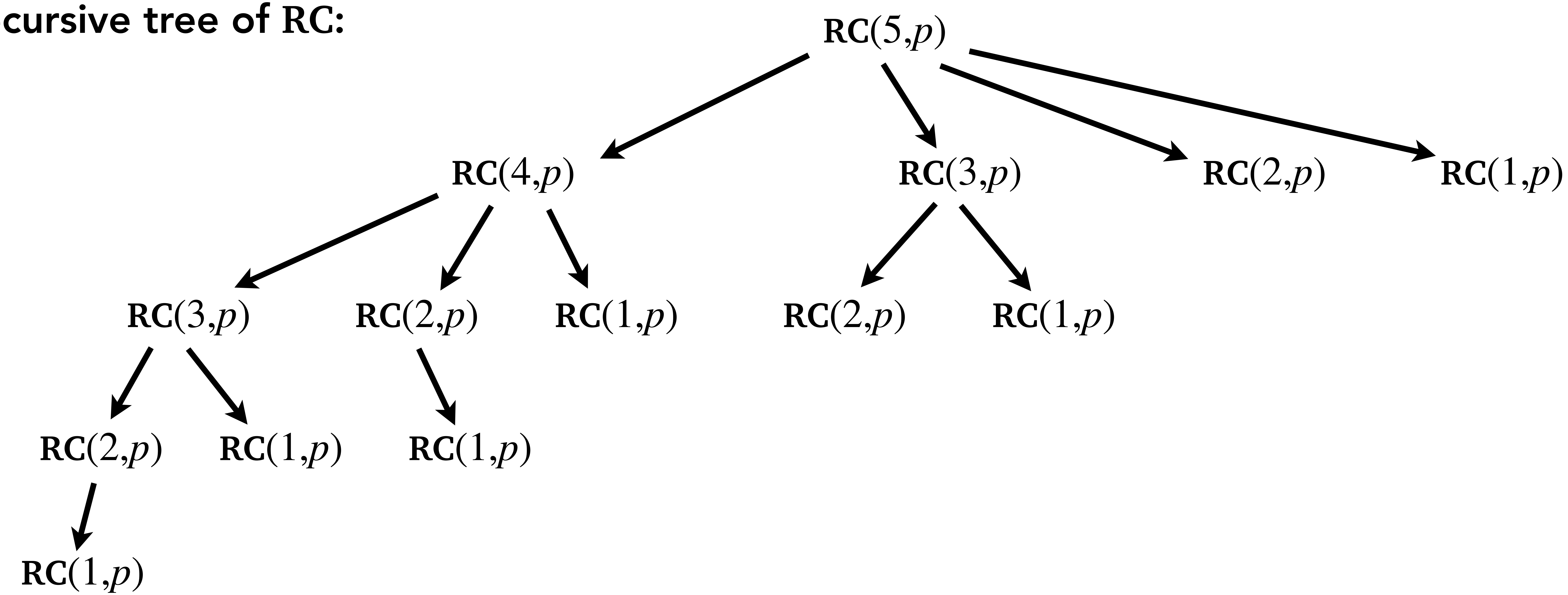

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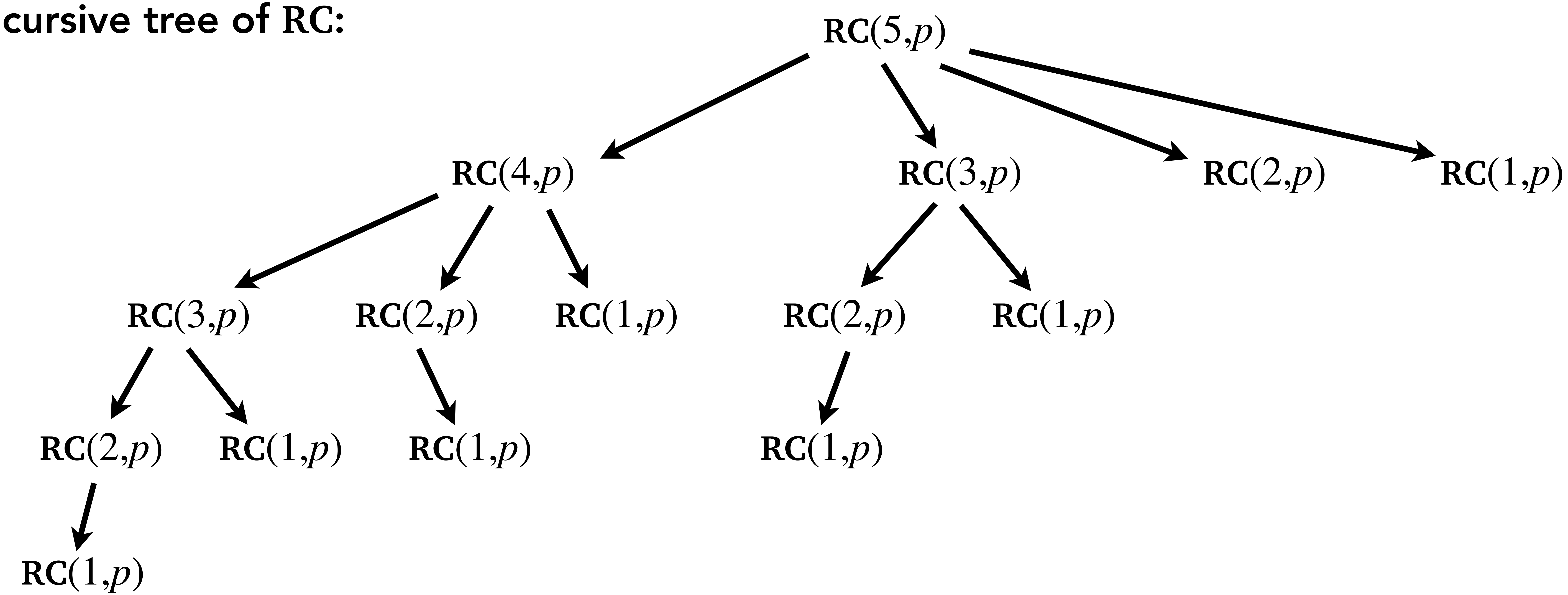

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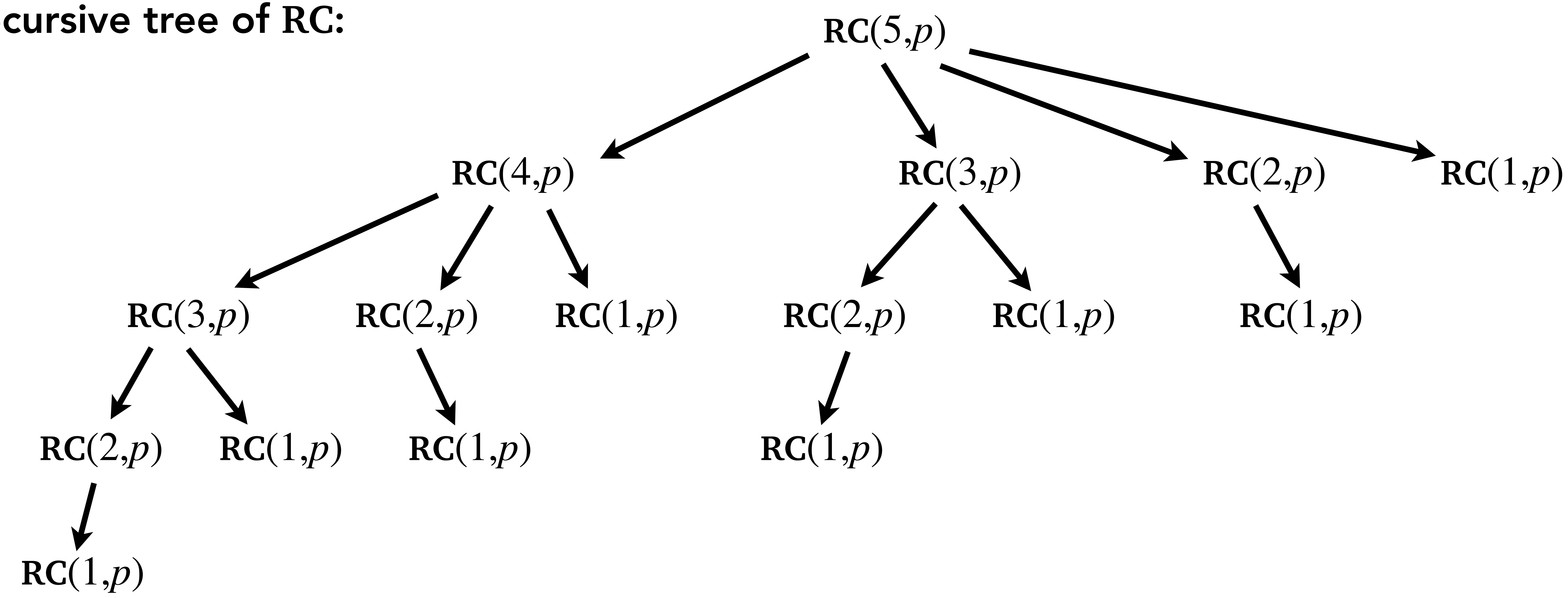

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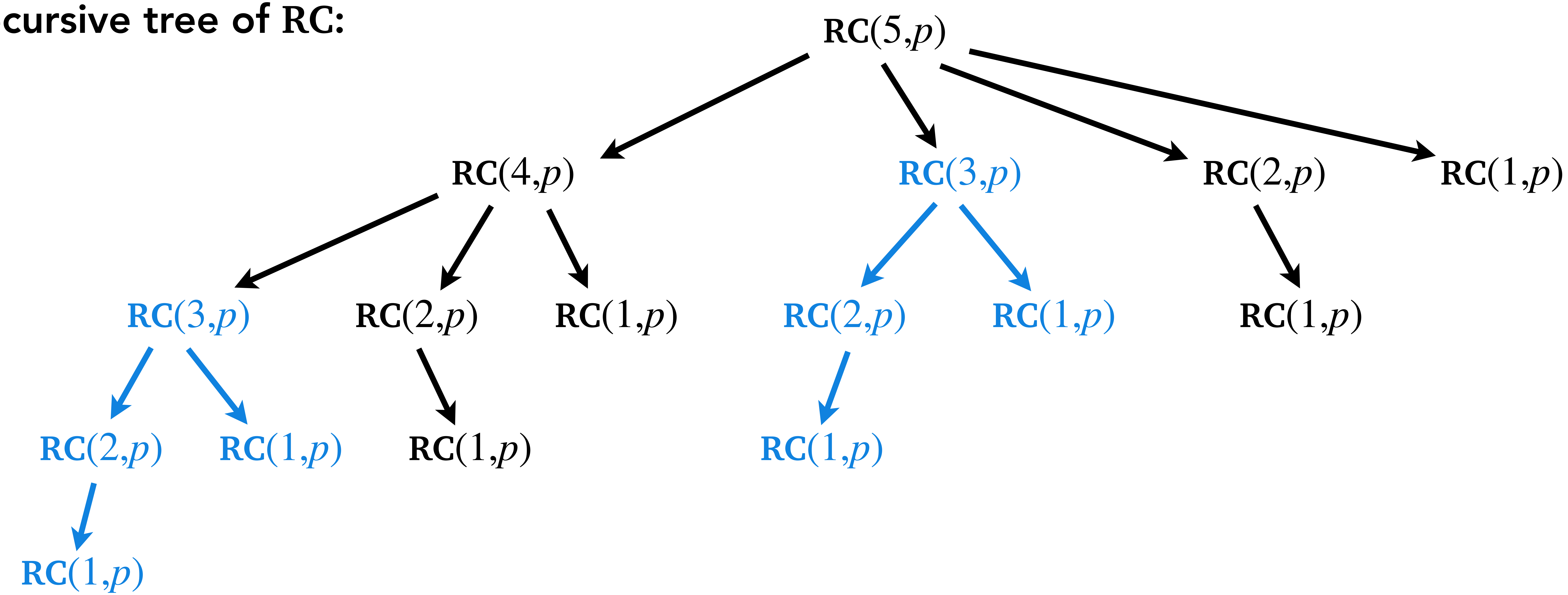

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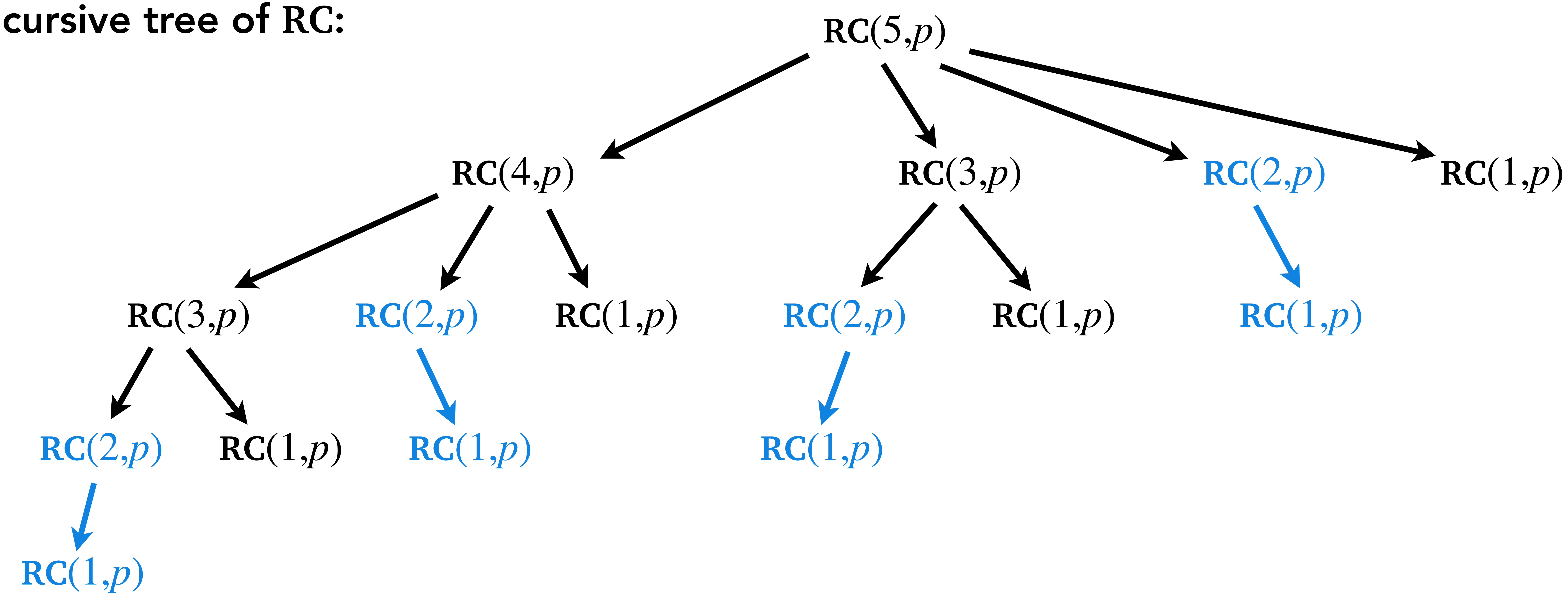

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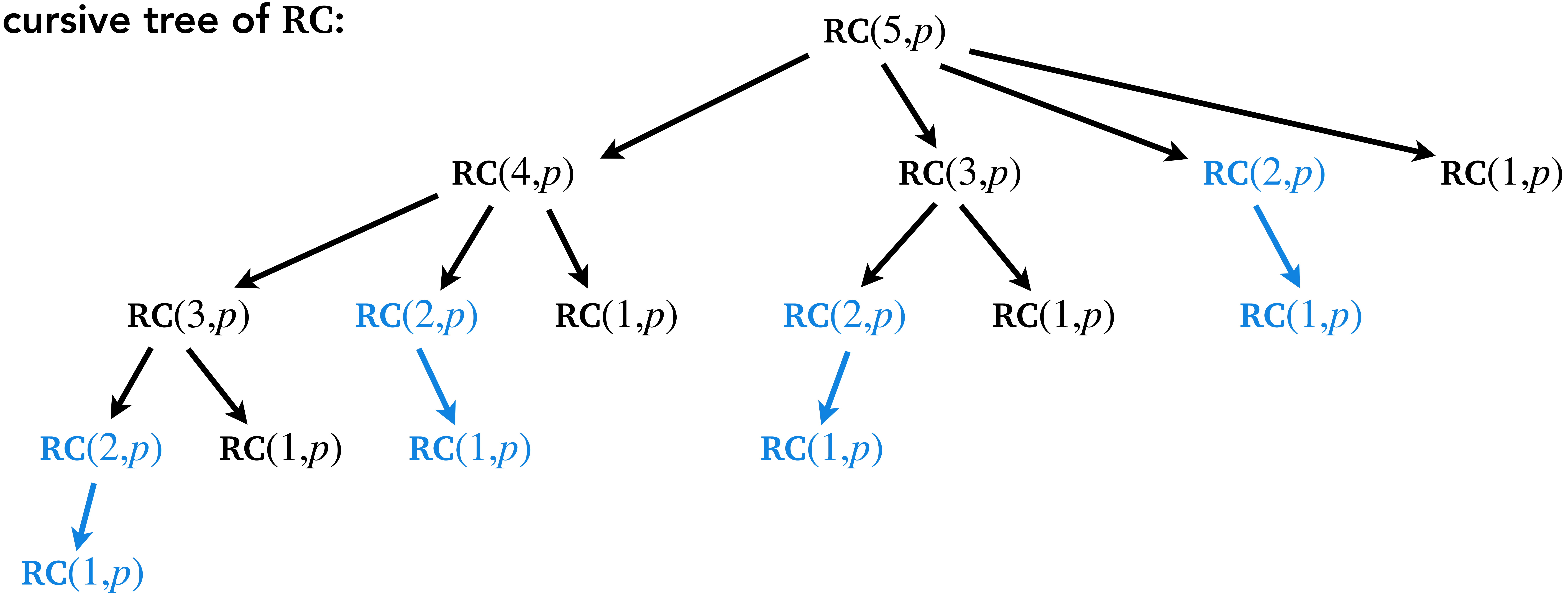

Observation: **RC**(3,*p*) and **RC**(2,*p*) is getting computed from scratch multiple times.

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1. **if** $profit[n] \neq -1$ ← Profit of n length rod is already computed
2. **return** $profit[n]$

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Dynamic Programming Enters

Idea: 1) Define an array $profit[1:n]$

2) Store the maximum profit obtainable from an i length rod in $profit[i]$ for future use after calculating it the first time.

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Let $T(n)$ denote the runtime of $\textbf{RC}(n, p)$

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Let $T(n)$ denote the runtime of $\mathbf{RC}(n, p)$

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```

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Cost of line 1,2,3, & 6

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Let $T(n)$ denote the runtime of $\mathbf{RC}(n, p)$

Then,

Cost of $\mathbf{RC}(n-1, p)$

$$T(1) = c$$

$$T(n) = \underbrace{c'}_{\text{Cost of line 1,2,3, \& 6}} + \Theta(n) + \overbrace{T(n-1)}^{\text{Cost of } \mathbf{RC}(n-1,p)}$$

Cost of line 1,2,3, & 6

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Time Complexity of RC

Let $T(n)$ denote the runtime of $\mathbf{RC}(n, p)$

Call to $\mathbf{RC}(n-1, p)$ computes $\mathit{profit}[1:(n-1)]$

Then,

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Call to $\mathbf{RC}(n-1, p)$ computes $profit[1:(n-1)]$

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Cost of $\mathbf{RC}(n-1, p)$

Cost of line 1,2,3, & 6

Cost of for loop from $i = 2$ to $n-1$ as $\mathbf{RC}(n-i, p)$ will return immediately.

```
profit[1 : n] = {p[1], -1, ..., -1}
RC(n, p):
1.  if profit[n] ≠ -1
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3.  profit[n] = p[n]
4.  for i = 1 to n - 1
5.      profit[n] = Max(profit[n], p[i] + RC(n - i, p))
6.  return profit[n]
```

Time Complexity of RC

Let $T(n)$ denote the runtime of $\mathbf{RC}(n, p)$

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$$= c' + \Theta(n) + c' + \Theta(n-1) + T(n-2)$$

Time Complexity of $\mathbf{RC}$

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Time Complexity of **RC**

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